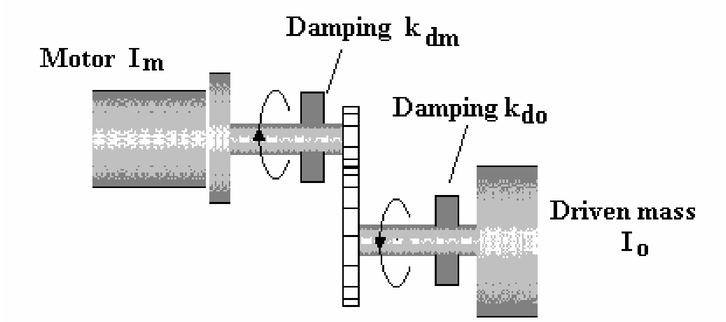
D225 Q8 1999

A motor produces a torque T that is related to angle of rotation θ by T = 500 sin θ over half a revolution and then repeated. The motor is connected to a load through a 3/1 reduction gear box. The moment of inertia of the motor is 0.2 kg m^2 and the load is 1 kg m^2.

(a) Construct a dynamically equivalent system referred to the motor shaft.
(b) Determine the inertia of the flywheel to be added to the motor shaft in order control the load speed at 250 rev/min ±1%.
(c) Calculate the maximum angular acceleration of the load during each revolution.

SOLUTION

(a) When a mass is rotated through a gear system, the affect of the inertia is dramatically altered. Consider a motor coupled to a load through a speed changing device such as a gear box. In this case there is no damping.



θ_m is the motor rotation and θ_o the output rotation. The gear ratio is $G_r = \theta_o/\theta_m$

Since this is a fixed number and is not a function of time, the speed and acceleration are also in the same ratio.

$G_r = \omega_o/\omega_m$ ω is the angular velocity

$G_r = \alpha_o/\alpha_m$ α is the angular acceleration.

The power transmitted by a shaft is given by Power = ωT. If there is no power lost, the output and input power must be equal so it follows that

$\omega_m T_m = \omega_o T_o$ hence $T_m = \omega_o T_o/\omega_m = G_r T_o$

Consider the inertia torque due the inertia on the output shaft I_o.

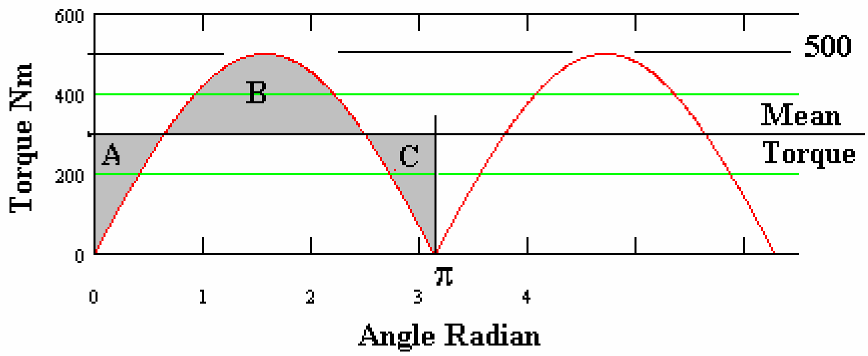
$T_o = I_o\alpha_o = I_o \alpha_m \times G_r$

$T_m = T_o \times G_r = I_o \alpha_m \times G_r^2$ This is the output inertia torque referred to the motor. In addition we have the inertia torque of the motor itself $I_m \alpha_m$. This mus be added so:

$T_m = I_m\alpha_m + I_o \alpha_m \times G_r^2$

$T_m = \alpha_m (I_m + G_r^2 I_o)$ and $(I_m + G_r^2 I_o)$ is the effective moment of inertia I_e referred to the motor shaft.

(b)



The coefficient of fluctuation of speed $\phi = \Delta\omega/\omega$ = 2% or 0.02 (for both motor and load)
The coefficient of fluctuation of energy β
β = max fluctuation in energy/work done per cycle βW = max fluctuation in energy
$\beta W = I\omega^2\phi$ (see tutorial on fly wheels and TM diagrams)
ω = mean angular velocity.
Motor speed = 750 rev/min $\omega = 2\pi \times 750/60 = 78.54$ rad/s
W = work done per cycle. In this case we will choose ½ revolution as the cycle since it is repeated every ½ revolution.

$$W = \int_0^{\pi} 500 \sin\theta \, d\theta = 500\left[\cos\theta\right]_0^{\pi} = 1000 \text{ J}$$

Mean Torque = W/base = $1000/\pi$ =318.31 Nm

The angle where the graph cuts the mean is $\sin^{-1}(318.31/500) = 39.54^o$ (0.69 rad) and $180^o - 39.54^o = 140.46^o$ (2.451 rad).

The area represented by B is found from: $W = \int_{0.69}^{2.45} 500 \sin\theta \, d\theta = 500\left[\cos\theta\right]_{0.69}^{2.45} = 771.178 \text{ J}$

It follows that areas A and B are both half this and equal to 385.59 J
The maximum fluctuation of energy is from (mean - 385.59) to (mean + 385.59) so $\beta W = 771.178$
$\beta = I\omega^2\phi/W$ so $I = \beta W/\omega^2\phi = 771.178/(78.54^2 \times 0.02) = 6.251$ kg m^2
$I_T = I_m + Gr^2 I_o + I$
$6.251 = 0.2 + (1/3)^2 \times 1 + I$
$6.251 = 0.311 + I$ $I = 5.94$ kg m^2

(c) $T = I\,\alpha$ α = angular acceleration of the motor.
The maximum torque is 500 N m so the maximum $\alpha = 500/6.251 = 80$ rad/s^2
The maximum acceleration of the load is reduced by the gear ratio of 3 so the maximum acceleration of the load is $80/3 = 26.7$ rad/s^2