D225 2003 Q7

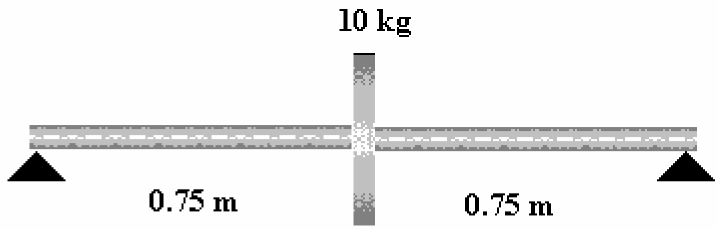
A 10 kg disc is mounted at the centre of a shaft 1.5 m long as shown.

Data $E = 209$ GPa $I = 2.68 \times 10^{-7}$ m^4 $M = 10$ kg $L = 1.5$ m δ = damping ratio

The log decrement for free transverse damped vibration is 0.628.

The deflection at the middle for a point load F is $y = FL^3/48EI$

(a) Show that the damping ratio δ is 0.1
(b) Determine the critical speed of the rotor.
(c) Calculate the amplitude of vibration at the middle if the centre of gravity is 1 mm from the centre and the shaft is revolved at 300 rev/min.



For a shaft with a point load at the middle the natural frequency of transverse vibrations are

$$\omega_n = \sqrt{\frac{48EI}{ML^3}} = \sqrt{\frac{48 \times 209 \times 10^9 \times 2.68 \times 10^{-7}}{10 \times 1.5^3}} = 282.244 \text{ rad/s}$$

Derivation $\omega_n = (k/M)^{1/2}$ for a simple spring mass system.
Given the deflection at the middle $y = FL^3/48EI$ stiffness $k = F/y = 48EI/ L^3$
$\omega_n = (k/M)^{1/2} = (48EI/ ML^3)^{1/2}$

log decrement is $\ln(x_1/x_2) = 0.628$

$$\ln\left(\frac{x_1}{x_2}\right) = 0.628 = \frac{2\pi\ \delta}{\sqrt{1 - \delta^2}}$$

$$0.1 = \frac{\delta}{\sqrt{1 - \delta^2}} \qquad 0.01 = \frac{\delta^2}{1 - \delta^2}$$

$0.01 - 0.01\delta^2 = \delta^2$ $0.01 = 1.01\delta^2$ $\delta^2 = 0.00990099$ $\delta = 0.0995$ round off 0.1

(b) The critical speed of the rotor corresponds to the damped resonant frequency of vibration.

$\omega_r = \omega_n (1 - \delta^2)^{1/2} = 282.244(1 - 0.1^2)^{1/2} = 280.83$ rad/s

Convert to rev/s and $N_r = (280.83/2\pi) = 44.695$ rev/s or 2681.7 rev/min

(c) $N = 3000$ rev/min $\omega = 2\pi N/60 = = 314.16$ rad/s eccentricity $e = 1$ mm

For a whirling shaft with no damping, the displacement is x given by

$$x = \frac{e}{\left(\frac{\omega_r}{\omega}\right)^2 - 1} = \frac{1}{\left(\frac{280.83}{314.16}\right)^2 - 1} = -5 \text{ mm}$$

Note the precise answer depends very much on how much rounding off is done and on a computer the answer is 9.84 mm

Using the formula given

$$x = \frac{F/k}{\sqrt{\left(1 - r^2\right)^2 + (2\delta r)^2}} \cos(\omega t + \theta)$$

This is a maximum when $\cos(\omega t+\phi) = 1$ and further the force is the centrifugal force.

$F = M\omega^2(x + e)$ $x + e$ is the radius to the centre of gravity.
$F/k = (M/k)\omega^2(x + e)$
Since $k/M = \omega_n^2$ then $F/k = r^2(x + e)$

$$x = \frac{r^2(x + e)}{\sqrt{\left(1 - r^2\right)^2 + (2\delta r)^2}}$$

$$\text{Let } p = \frac{r^2}{\sqrt{\left(1 - r^2\right)^2 + (2\delta r)^2}}$$

$x = p(x + e)$ $x(1 - p) = pe$ $x = \frac{pe}{1 - p}$ This gives an answer of 2.45 mm

Evaluating with D = 0 we get 1.24 mm and this should agree with the first answer .. something wrong but I cant spot it.