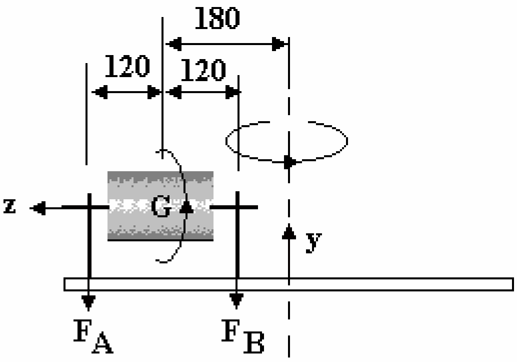
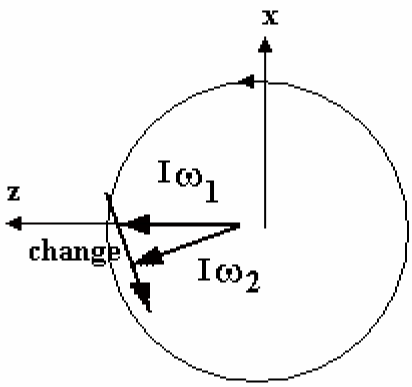
D25 1999 Q5

GYROSCOPE

A motor has a mass of 10 kg and is mounted on a platform as shown. The rotor has a mass of 2.5 kg and turns at 1750 rev/min anticlockwise viewed from the back. The radius of gyration is 45 mm. The platform is rotated at 50 rev/min anticlockwise viewed from above. Calculate the reactions at the motor mounts A and B.



Viewed from above the change in angular momentum is as shown. This means a reaction torque will be produced about the x axis. The magnitude is $T = I\,\omega_z\omega_y$



$I = m k^2 = 2.5 \times 0.045^2 = 5.062 \times 10^{-3}$ kg m^2

$\omega_z = 2\pi N_z/60 = 2\pi \times 1750/60 = 183.26$ rad/s

$\omega_y = 2\pi N_y/60 = 2\pi \times 50/60 = 5.236$ rad/s

Gyroscopic torque $= T = 5.062 \times 10^{-3} \times 183.26 \times 5.236 = 4.858$ N m

Assume this is a force F acting at the centre of gravity. F = T/0.18 = 26.987 N

The reaction torque is the opposite vector to the change shown on diagram so it exerts an upwards force.

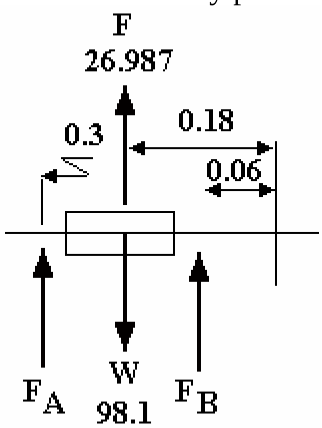
Weight of motor = M g = 10 g = 98.1 N and this is down (minus).

Forces acting on the motor in vertical direction are F, F_A F_B and the weight Mg must all add up to zero.

$$26.87 + F_A + F_B - 98.1 = 0$$

$$F_A = 71.23 - F_B$$

Moments about any point must add to zero so about the centre of the platform we have



$(F \times 0.18) - (W \times 0.18) + (F_A \times 0.3) + (F_B \times 0.06) = 0$

$(26.987 \times 0.18) - (W \times 0.18) + (F_A \times 0.3) + (F_B \times 0.06) = 0$

$(26.987 \times 0.18) - (98.1 \times 0.18) + (F_A \times 0.3) + (F_B \times 0.06) = 0$

$(F_A \times 0.3) + (F_B \times 0.06) = 12.8$

$\{(71.23 - F_B) \times 0.3\} + (F_B \times 0.06) = 12.8$

$21.369 - 0.3F_B + 0.06\,F_B = 12.8$

$8.569 = 0.24\,F_B$

$F_B = 35.70$ N

$F_A = 71.23 - 35.70 = 35.5$ N