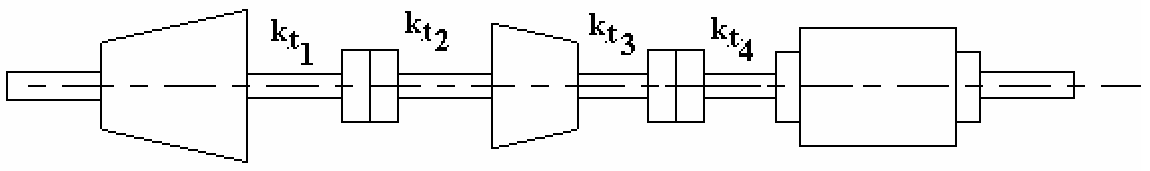
Q4 1999 D225 DYNAMICS OF MECHANICAL SYSTEMS

A gas turbine is connected to a compressor and a generator through shafts and couplings with stiffness and moment of inertia as shown on the diagram.



Severe torsional observations occur when running at the normal speed of 50 rev/s. Neglect the inertia of the shafts and determine the fundamental natural frequency and the mode shape. Use Holzer's method to do this.

SOLUTION

There are five moments of inertia and four shaft sections. From left to right these are

$I_1 = 10$ $I_2 = 1$ $I_3 = 5$ $I_4 = 1$ $I_5 = 5$ $k_{t1} = k_{t2} = 2 \times 10^6$ $k_{t3} = k_{t4} = 1 \times 10^6$

The equations to be solved using Holzers method are

$y_1 = 1$

$$y2 := y1 - \frac{\omega^2 \cdot I1 \cdot y1}{k1}$$

$$y3 := y2 - \frac{\omega^2}{k2} \cdot (I1 \cdot y1 + I2 \cdot y2)$$

$$y4 := y3 - \frac{\omega^2}{k3} \cdot (I1 \cdot y1 + (I2 \cdot y2 + I3 \cdot y3))$$

$$y5 := y4 - \frac{\omega^2}{k4} \cdot (I1 \cdot y1 + (I2 \cdot y2 + I3 \cdot y3) + I4 \cdot y4)$$

$$T1 := \omega^2 \cdot I1 \cdot y1$$

$$T2 := T1 + \omega^2 \cdot I2 \cdot y2$$

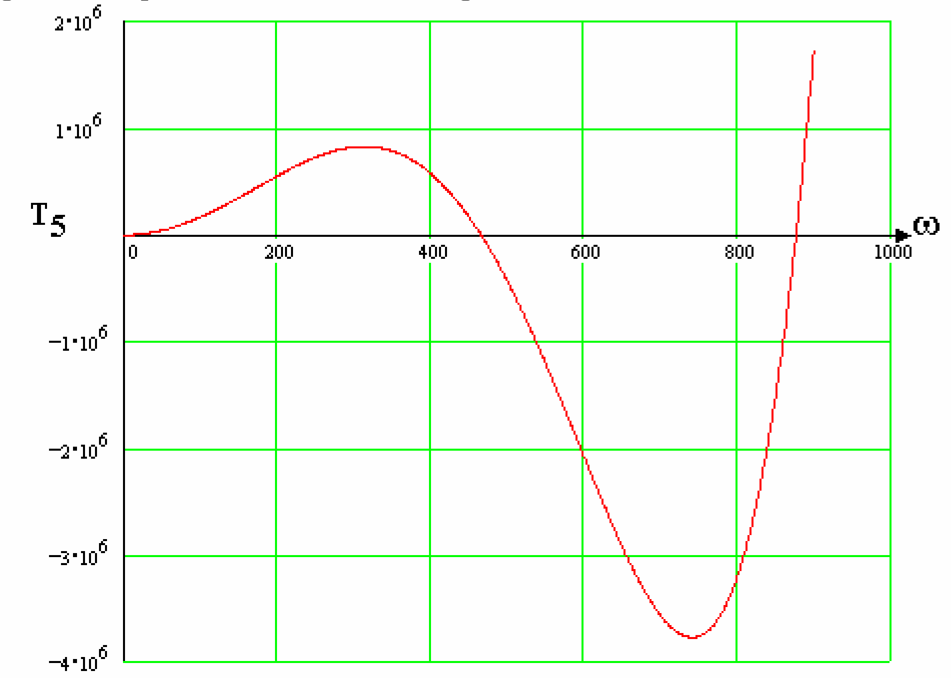
$$T3 := T2 + \omega^2 \cdot I3 \cdot y3$$

$$T4 := T3 + \omega^2 \cdot I4 \cdot y4$$

$$T5 := T4 + \omega^2 \cdot I4 \cdot y4$$

These must evaluated for various values of ω and the value when $T_5 = 0$ found.

There should be a clue that this might be around 50 rev/s or 50 x 2 x π = 314 rad/s or some multiple. However solving on a computer shows the following.



Finding the correct values by method would probably take all day with a calculator so one wonders what the examiner had in mind.

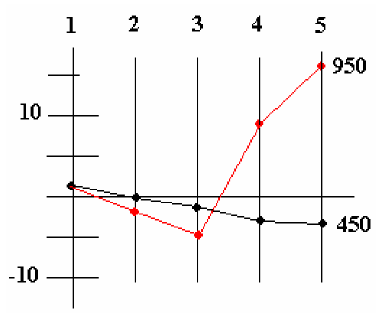
Evaluating in steps of 100 rad/s might have been advised.

ω	0	100	200	300	400	500	600	700	800	900
T_5 x 10^{-6}	0	0.16	0.55	0.82	0.57	-0.43	-2.08	-3.56	-3.2	1.73
Y_2	1.0	0.95	0.8	0.55	0.2	-0.25	-0.8	-1.45	-2.2	-3.05
Y_3	1.0	0.89	0.58	0.07	-0.62	-1.47	-2.46	-3.54	-4.7	-5.86
Y_4	1.0	0.74	0.035	-0.90	-1.75	-2.07	-1.35	0.95	5.34	12.26
Y_5	1.0	0.56	-0.52	-1.81	-2.61	-2.15	0.25	4.98	11.95	20.45

There is a critical frequency between 400 and 500 rad/s and between 800 and 900 rad/s. These could be narrowed down further but in an exam it might be reasonable to guess at 450 rad/s and 950 rad/s. Note that the higher one is around three times the running speed so his might account for the vibration.

To find the node, we need the deflections at the critical speed. Guestimate the values and plot.

ω	450	850
Y_1	1	1
Y_2	0	-2.7
Y_3	-1	-5
Y_4	-1.9	9
Y_5	-2.4	16



At the lower speed there is a node close to the first coupling. At the higher speed there are two nodes, one in the first section and one in the third section of shaft.