DYNAMICS OF MECHANICAL SYSTEMS SOLUTIONS Q1 1999

A machine of mass 70 kg is mounted on springs of stiffness 1 MN/m and dampers with a damping ratio of 0.2. A piston on the machine has a mass of 1.8 kg and reciprocates up and down harmonically with a stroke of 75 mm and speed of 3000 rev/min.

Calculate (i) the amplitude of vibration for the machine.
(ii) the phase angle of the motion with respect to the exciting force.
(iii) the force transmitted to the foundations.
(iv) the phase angle of the transmitted force with respect to the exciting force.

SOLUTION

Data $M = 70$ kg $k = 10^6$ N/m $m = 1.8$ kg $\xi = 0.2$

Amplitude of motion $a = 75/2 = 37.5$ mm Speed = 3000 rev/min $\omega = 2\pi N/60 = 314.159$ rad/s
Natural Frequency of System $\omega n = \sqrt{(k/M)} = 119.523$ rad/s
Motion is $x = A_1 \cos(\omega t)$ vel $= -A_1 \omega \sin(\omega t)$ acc $= -A_1 \omega^2 \cos(\omega t) = -\omega^2 x$
Inertia Force = m acc = $-m\,\omega^2\,x$
$Fo = m\,A_1\,\omega^2 = 6.662$ kN
Check $Fo := m \cdot \omega^2 \cdot A_1$ $Fo = 6.662 \cdot 10^3$

$$A := \frac{Fo}{M} \cdot \sqrt{\frac{1}{\left(\omega n^2 - \omega^2\right)^2 + (2 \cdot \zeta \cdot \omega \cdot \omega n)^2}}$$

A = amplitude of machine = 1.11 mm

$$\phi := \operatorname{atan}\left(\frac{2 \cdot \zeta \cdot \omega \cdot \omega n}{\omega n^2 - \omega^2}\right)$$

ϕ = phase angle w.r.t. disturbing force = - 10.09 degrees
Cc = critical damping coefficient $Cc = \sqrt{(4\,M\,k)} = 16730$ Nm/s
c = actual damping coefficient $c = \xi Cc = 3\,347$ Ns/m
Fs = spring force $= k\,A = 1.11$ kN
Fd = damping force $= c\,A\,\omega = 1.167$ kN
FT = Transmitted force $= \sqrt{(Fs^2 + Fd^2)} = 1.611$ kN
ϕT = phase angle $= \phi - \tan^{-1}(Fd/Fs) = -56.52$ degrees
Check with formulae given in question $r = \omega/\omega n$

$$A := \frac{\frac{Fo}{k}}{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]^{\frac{1}{2}}}$$

A = 1.11 mm