ENGINEERING COUNCIL EXAM MATHEMATICS C101
SOLUTIONS TO EXAM PAPER 2004

Q9

The lifespan in hours of a mass produced light optical device is normally distributed and has a mean of 1400 with a standard deviation of 300.

a) What is the probability of one taken at random having a lifespan between 1400 and 1850 hours?

b) What is the percentage that will last longer than 2100 hours?

c) If the guarantee is for 1000 hours, what percentage will fail to meet the guarantee?

d) What lifespan should be guaranteed if 95% must obtained?

SOLUTION

Convert the variables to a z score using $z = \dfrac{x - \bar{x}}{\sigma}$

a) Convert 1850 hours $z = \dfrac{1850 - 1400}{300} = 1.5$

Area under the standard normal curve - depends on the table you are using. From the one supplied with the exam paper at z = 1.5 Area = 0.4332. This is the area from z = 0 to z = 1.5

For 1400 hours z = 0 hence the area between z = 0 and z = 1.5 is 0.4332

The probability is hence 43.3%

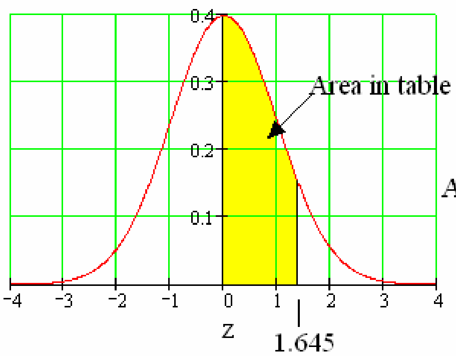
b) Convert 2100 hours $z = \dfrac{2100 - 1400}{300} = 2.333$

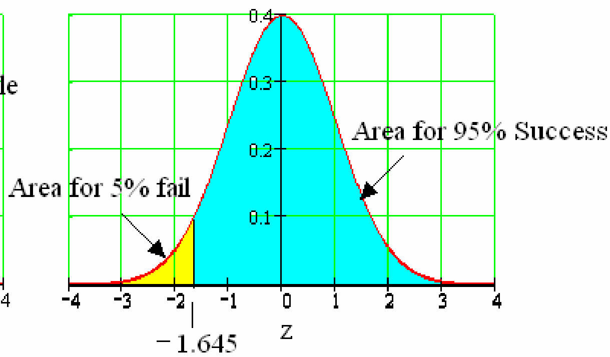
From the tables the area between z = 0 and z = 2.333 is 0.4902. The area representing all larger figures is 0.5 – 0.4902 = 0.0098 so the % that will last longer is 0.98%

c) Convert 1000 hours $z = \dfrac{1000 - 1400}{300} = -1.333$

The area between z = 0 and z = 1.333 is the same as between 0 and 1.333 and is 0.4087
The area between z = 0 and z = -1.333 is 0.5 – 0.4087 =0.0913 hence the % that will fail is 9.13%

d) For 95% success we require 5% to fail. 5% must give the area between z = -∞ and z.





This will be the same as the area between z and z = ∞.
The area in the table will be 0.95 – 0.5 = 0.45.
From the table this is 1.645 so we the required z is -1.645

$z = -1.645 = \dfrac{t - 1400}{300}$ hence t = 906.5 hours