# ENGINEERING COUNCIL EXAM MATHEMATICS C101
# SOLUTIONS TO EXAM PAPER 2004

Q4a

A hole of diameter D = 15 cm and height H = 15 cm is to be cut in a block of metal. The cutting process has a maximum error of 0.002 cm in the diameter and 0.001 cm in the height. Use partial differentiation to determine the maximum error in the volume of the cavity.

**SOLUTION**

Volume = area x height = $\pi D^2/4$ x H $\qquad V = \pi D^2 H/4$

Partial differentiation first keep H constant $\frac{\partial V}{\partial D} = \frac{\pi DH}{2}$

Next keep D constant $\frac{\partial V}{\partial H} = \frac{\pi D^2}{4}$

Error in the volume is $\delta V = \frac{\partial V}{\partial D}\delta D + \frac{\partial V}{\partial H}\delta H = \frac{\pi DH}{2}\delta D + \frac{\pi D^2 H}{2}\delta H$

$$\delta V = \frac{\pi \text{ x } 15 \text{ x } 10}{2} \text{ x } 0.002 + \frac{\pi \text{ x } 15^2 \text{ x } 10}{2} \text{ x } 0.001 = 0.648 \text{ cm}^3$$

The answer may be verified by calculating the volume with the minimum and then the maximum dimensions.

$V_1 = \pi$ x $15^2$ x 10/4 = 1767.146 cm$^3$
$V_2 = \pi$ x $15.001^2$ x 10.002/4 = 1767.794 cm$^3$
$\Delta V$ = 1767.794 - 1767.146 = 0.648 cm$^3$

Q4(b)
The ideal gas law is p$v$ = RT where p is the pressure, $v$ the specific volume, T the absolute Temperature and R the characteristic gas constant. By finding the differential dp derive a differential equation for an isobar (a curve of constant p)

**SOLUTION**

p$v$ = RT rearrange to p = RT/$v$ and partial differentiation yields:

$$\frac{\partial p}{\partial v} = -\frac{RT}{v^2} \qquad \frac{\partial p}{\partial T} = \frac{R}{v}$$

$$dp = \frac{\partial p}{\partial T}dT + \frac{\partial p}{\partial v}dv$$

$$dp = \frac{R}{v}dT - \frac{RT}{v^2}dv$$

On an isobar dp = 0 everywhere so

$$\frac{R}{v}dT = \frac{RT}{v^2}dv$$

$$\frac{dT}{T} = \frac{dv}{v}$$