ENGINEERING COUNCIL EXAM MATHEMATICS C101
SOLUTIONS TO EXAM PAPER 2004

Q3a. A thick cylinder has an internal radius r_o and external radius r_1. The radial compressive stress p at distance r from the axis of the cylinder is given by the differential equation

$$r\frac{dp}{dr} = 2(A-p) \text{ where A is a constant}$$

Solve the equation if $p = p_o$ at $r = r_o$

SOLUTION

The variables are separable. $\frac{dp}{(A-p)} = \frac{2dr}{r}$

We can make a substitution (A - p) = X

Differentiate –dp = dX and substitute

$$-\frac{dX}{X} = \frac{2dr}{r}$$

Now integrate $-\ln(X) = 2\ln(r) + C$

Substitute back $-\ln(A-p) = 2\ln(r) + C$

Boundary condition $p = p_o$ at $r = r_o$

$$-\ln(A-p_o) = 2\ln(r_o) + C$$

$$C = -\frac{\ln(A-p_o)}{2\ln(r_o)}$$

The equation is now

$$-\ln(A-p) = 2\ln(r) - \frac{\ln(A-p_o)}{2\ln(r_o)}$$

$$\ln(\frac{1}{A-p}) = \ln(r^2) - \frac{\ln(A-p_o)}{\ln(r_o^{\,2})}$$

Take anti-logs

$$\frac{1}{A-p} = r^2 - antilog\left(\frac{\ln(A-p_o)}{\ln(r_o^{\,2})}\right)$$

$$\frac{1}{A-p} = r^2 - K$$

$$A-p = \frac{1}{r^2-K}$$

$$p = A - \frac{1}{r^2-K}$$

I have some doubt about this solution anyone able to offer a better one please contact freestudy.

Q3(b) A cylindrical tank is receiving and discharging water at the same time. Initially the tank is empty and at time t the depth is h. h and t are related by the equation

$$\frac{dh}{dt} + kh = kh_o e^{-kt}$$ where k and h_o are constants

Find the depth oif water as a function of t and sketch the graph of h against t.

SOLUTION

Compare $\frac{dh}{dt} + kh = kh_o e^{-kt}$ and $\frac{dh}{dt} + hP(t) = Q(t)$

$P(t) = k$ $\quad Q(t) = kh_o e^{-kt}$ $\quad IF = e^{\int P(t)dt} = e^{\int kdt} = e^{kt}$

$$h(t) = \frac{1}{e^{kt}} \int kh_o e^{-kt} e^{kdt} dt = \frac{1}{e^{kt}} \int kh_o e^{o} dt = e^{-kt} \int kh_o dt$$

$h(t) = e^{-kt} kh_o t + C$ $\quad$ When t = 0, h = 0 $\quad 0 = 0 + C$ $\quad$ C = 0

$h = e^{-kt} kh_o t$

The graph is typically as shown which does not make a lot of sense as it appears to fill and empty.